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FUZZY FLOYD'S ALGORITHM TO FIND SHORTEST ROUTE BETWEEN NODES UNDER UNCERTAIN ENVIRONMENT

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ABSTRACT

Normally, Floyd's – Warshall algorithm is used to find shortest route between any two nodes in the network. In this article, a generalized Floyd's algorithm is projected under the effect of uncertain environment. Two key issues need to be addressed in shortest route problem with fuzzy parameters. One is to determine the addition of weights for two edges; other is comparison of weights between two different paths which are represented by fuzzy numbers. To work out this problem, the graded mean integration representation of fuzzy numbers is used to improve the classical Floyd's algorithm. An illustration of a transportation network model is used to illustrate proposed method.

KEYWORDS: Floyd's Algorithm, Fuzzy Numbers, Graded Mean Representation

1. INTRODUCTION

In many real management systems, which are in terms of network model, it is necessary to provide the route between any two nodes which carries minimum weight. In a transportation network model, the weights on edges are assumed to represent transportation time or cost, rather than geographical distances. The notation describing an acyclic transportation network model is (N,E), where $N=\{1,2,3,....,n\}$ is the set of nodes and $E\subseteq N\times N$ is the set of directed edges. Each edge is denoted by an ordered pair of vertices (i,j), where $i,\ j\in N$. It is assumed that there is maximum one directed edge (i,j) from i to j. In classical graph theory it is assumed, weights on the edges are always a crisp values for the given parameters, however in practical situations parameters are not always precise (e.g. transportation cost, transshipment times, capacities, demands, etc.) [1]. Y. Deng et. al had discussed Dijkstra's Algorithm under uncertain enviornment for shortest path problem [2] and provided an up to date citation to discuss wide use of Fuzzy set theory to handle vague information in many fields such as environmental assessment, differentiating effectiveness of drugs, decision making [2].

In finding the shortest route between any two nodes in an uncertain environment, an appropriate modeling approach is to make use of fuzzy numbers. As a result many researchers have paid attention to the fuzzy shortest route problem.

Normally Floyd's Algorithm is used to find shortest route between any two nodes in a network model where edge weights are given as crisp numbers. However, due to the reason that many optimization methods for crisp numbers can not be applied directly for fuzzy numbers, few modifications must be done before applying classical methods. One can transform the fuzzy numbers in to crisp one using some standard defuzzification function [3]. There are mainly two issues that need to be solved while applying the Floyd's algorithm in a fuzzy environment. First, is to summing operations of fuzzy numbers and second is ranking and comparison of fuzzy numbers, which is still an open problem in fuzzy set theory

research fields.

The canonical representation of operations on fuzzy numbers that are based on the graded mean integration representation method leads to the result that multiplication and addition of two fuzzy numbers can be represented as crisp number [4]. This method is widely used in many applications such as multi-criteria decision making [5-9], risk evaluation [10], portfolio selection [11], evaluation of airline service quality [12].

In this article, the classical Floyd's algorithm is generalized based on the canonical representation of operations on triangular or trapezoidal fuzzy numbers to handle the fuzzy shortest route problem. Compared with existing methods, the proposed method is more efficient due to the fact that the summing operation and the ranking of fuzzy numbers can be done in an easy and straight manner. Here, Section 2 gives a short introduction to the basic theory used in our proposed method, including fuzzy set theory and the Floyd's algorithm. Section 3 develops the proposed method in detail. In Section 4, an illustration of a transportation network model is discussed to find shortest route between any two nodes under fuzzy environment to discuss the efficiency of proposed model. Section 5 deals with conclusion.

2. PRELIMINARIES

2.1. Fuzzy Numbers

Definition 2.1

Fuzzy Set: Let X be a universe of discourse. Where $\tilde{A}$ is a fuzzy subset of X ; and for all $x \in X$, there is a number $\mu_{\tilde{A}}(x) \in [0, 1]$ which assigned to represent the membership degree of x in $\tilde{A}$, and is called the membership function of $\tilde{A}$ [13].

Definition 2.2

Fuzzy number: A fuzzy number $\tilde{A}$ is normal and convex subset of X [13].

Here normality means: $\exists x \in R, \ni \underset{x}{\vee} \mu_{\tilde{A}}(x) = 1$ (2.1)

And convexity means:

$$\forall x_1 \in X, x_2 \in X, \forall \alpha \in [0, 1], \mu_{\tilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad (2.2)$$

Definition 2.3

Triangular Fuzzy Number: A triangular fuzzy number $\tilde{A}$ can be defined as a triplet (a, b, c), where membership function can be defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \\ 0 & x > c \end{cases} \quad (2.3)$$

Graphically triangular fuzzy number is defined in Figure 1.

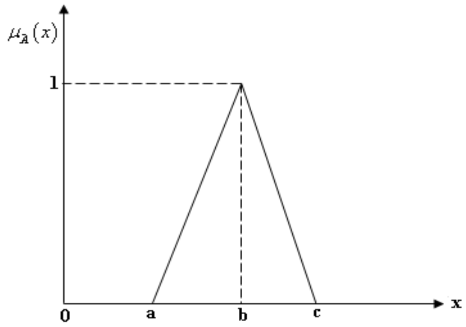

Figure 1: Triangular Fuzzy Number

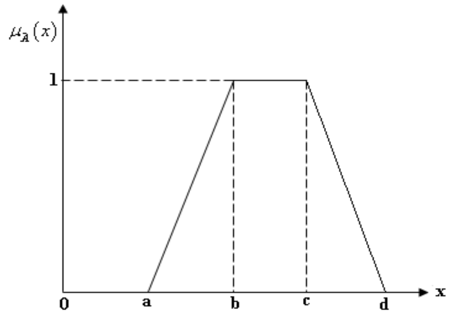

Figure 2: Trapezoidal Fuzzy Number

Definition 2.4

Trapezoidal Fuzzy Number: A trapezoidal fuzzy number $\tilde{A}$ can be defined as $\tilde{A}=(a, b, c, d)$, where membership function can be defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x<a \\ \dfrac{x-a}{b-a}, & a\le x\le b \\ 1, & b\le x\le c \\ \dfrac{d-x}{d-c}, & c\le x\le d \\ 0 & x>d \end{cases} \qquad (2.4)$$

Graphically triangular fuzzy number is defined in Figure 2.

2.2. The Fuzzy Arithmetical Operations

Fuzzy arithmetical operations under Function Principal are defined as follows [14].

Suppose $\tilde{A}=(a_1, a_2, a_3, a_4)$ and $\tilde{B}=(b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then

(1) Addition of $\tilde{A}$ and $\tilde{B}$ is defined as

$\tilde{A}\oplus\tilde{B}=(a_1+b_1,\ a_2+b_2,\ a_3+b_3,\ a_4+b_4)$, where $a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4$ are any real numbers.

(2) Multiplication of $\tilde{A}$ and $\tilde{B}$ is defined as

$\tilde{A}\otimes\tilde{B}=(c_1, c_2, c_3, c_4)$, where $T=\{a_1b_1,\ a_1b_4,\ a_4b_1,\ a_4b_4\}$, $T_1=\{a_2b_2,\ a_2b_3,\ a_3b_2,\ a_3b_3\}$, $c_1=\min T,\ c_2=\min T_1,\ c_3=\max T_1,\ c_4=\max T$. And $\tilde{A}\otimes\tilde{B}=(a_1b_1,\ a_2b_2,\ a_3b_3,\ a_4b_4)$, if $a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4$ are any positive real numbers.

(3) $-\tilde{B}=(-b_4, -b_3, -b_2, -b_1)$, then Subtraction of $\tilde{A}$ and $\tilde{B}$ is defined as

$\tilde{A}\ominus\tilde{B}=(a_1-b_4,\ a_2-b_3,\ a_3-b_2,\ a_4-b_1)$, where $a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4$ are any real numbers.

(4) $1/\tilde{B}=\tilde{B}^{-1}=\left(1/b_4,1/b_3,1/b_2,1/b_1\right)$, where b_1,b_2,b_3,b_4 are all positive real numbers.

If $a_1,b_1,a_2,b_2,a_3,b_3,a_4,b_4$ are all nonzero positive real numbers, then the Division of $\tilde{A}$ and $\tilde{B}$ is defined as $\tilde{A}\oslash\tilde{B}=\left(a_1/b_4,a_2/b_3,a_3/b_2,a_4/b_1\right)$

(5) Let $\alpha\in R$, then

$$\begin{cases}(i)\ \alpha\geq 0,\ \ \alpha\otimes\tilde{A}=\left(\alpha a_1,\alpha a_2,\alpha a_3,\alpha a_4\right)\\(ii)\ \alpha<0,\ \ \alpha\otimes\tilde{A}=\left(\alpha a_4,\alpha a_3,\alpha a_2,\alpha a_1\right)\end{cases}$$

2.3. Canonical Representation of Operations on Fuzzy Numbers

In this article, the canonical representation of operations on triangular fuzzy numbers which are based on the graded mean integration representation method [4] is used to obtain a simple fuzzy shortest route algorithm.

Definition 2.5

Given a triangular fuzzy number $\tilde{A}=\left(a_1,\ a_2,\ a_3\right)$, the graded mean integration representation of triangular fuzzy number $\tilde{A}$ is defined as:

$$P\left(\tilde{A}\right)=\frac{1}{6}\left(a_1+4\times a_2+a_3\right) \tag{2.5}$$

Let $\tilde{A}=\left(a_1,\ a_2,\ a_3\right)$ and $\tilde{B}=\left(b_1,\ b_2,\ b_3\right)$ be two triangular fuzzy numbers. By applying equation (2.5), the graded mean integration representation of triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be obtained respectively as follows

$$P\left(\tilde{A}\right)=\frac{1}{6}\left(a_1+4\times a_2+a_3\right)$$

$$P\left(\tilde{B}\right)=\frac{1}{6}\left(b_1+4\times b_2+b_3\right)$$

The representation of addition operation on triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as:

$$P\left(\tilde{A}\oplus\tilde{B}\right)=P\left(\tilde{A}\right)\oplus P\left(\tilde{B}\right)=\frac{1}{6}\left(a_1+4\times a_2+a_3\right)+\frac{1}{6}\left(b_1+4\times b_2+b_3\right) \tag{2.6}$$

The representation of multiplication operation on triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as:

$$P\left(\tilde{A}\otimes\tilde{B}\right)=P\left(\tilde{A}\right)\times P\left(\tilde{B}\right)=\frac{1}{6}\left(a_1+4\times a_2+a_3\right)\times\frac{1}{6}\left(b_1+4\times b_2+b_3\right) \tag{2.7}$$

Definition 2.6

Given a trapezoidal fuzzy number $\tilde{A}=\left(a_1,\ a_2,\ a_3,a_4\right)$, the graded mean integration representation of trapezoidal fuzzy number $\tilde{A}$ is defined as:

$$P(\tilde{A}) = \frac{1}{6}(a_1 + 2\times a_2 + 2\times a_3 + a_4) \tag{2.8}$$

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. By applying equation (2.8), the graded mean integration representation of trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be obtained respectively as follows

$$P(\tilde{A}) = \frac{1}{6}(a_1 + 2\times a_2 + 2\times a_3 + a_4)$$

$$P(\tilde{B}) = \frac{1}{6}(b_1 + 2\times b_2 + 2\times b_3 + b_4)$$

Similar to the triangular fuzzy number, the graded mean integration representation of operations on trapezoidal fuzzy numbers can be also obtained.

The representation of addition operation on trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as:

$$P(\tilde{A}\oplus\tilde{B}) = P(\tilde{A})\oplus P(\tilde{B}) = \frac{1}{6}(a_1 + 2\times a_2 + 2\times a_3 + a_4) + \frac{1}{6}(b_1 + 2\times b_2 + 2\times b_3 + b_4) \tag{2.9}$$

The representation of multiplication operation on trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as:

$$P(\tilde{A}\otimes\tilde{B}) = P(\tilde{A})\times P(\tilde{B}) = \frac{1}{6}(a_1 + 2\times a_2 + 2\times a_3 + a_4)\times\frac{1}{6}(b_1 + 2\times b_2 + 2\times b_3 + b_4) \tag{2.10}$$

For example, see Figure 3 and Table 1. From node 1 to 4, there are two routes. One is 1→2→4 and second is 1→3→4. The use of canonical representation of addition operation on fuzzy weights in shortest route finding problem can be used as follows:

$$\begin{aligned} weight_1(1,4) &= weight(1,2)\oplus weight(2,4) \\ &= (1,2,3,5)\oplus(2,5,6,7) \\ &= \frac{1}{6}(1+2\times2+2\times3+5)+\frac{1}{6}(2+2\times5+2\times6+7) \\ &= \frac{47}{6}\approx 7.833333 \end{aligned}$$

For the second route

$$\begin{aligned} weight_1(1,4) &= weight(1,3)\oplus weight(3,4) \\ &= (5,7,10,11)\oplus(3,6,7,8) \\ &= \frac{1}{6}(5+2\times7+2\times10+11)+\frac{1}{6}(3+2\times6+2\times7+8) \\ &= \frac{87}{6} \\ &= 14.5 \end{aligned}$$

Since 7.833333 < 14.5, one can easily verify that the first route is better than the second. From above example one can observe a benefit of using canonical representation of addition operation is that its results in to a crisp number. The decision making can be easily obtained without the process of ranking fuzzy numbers, commonly used in many other fuzzy shortest path problems. This is very advantageous in the Floyd's algorithm under fuzzy environment.

2.4 Floyd's Algorithm

Floyd's algorithm determines shortest route between any two nodes in the network [15]. The algorithm represents an n - nodes network as a square matrix with n rows and n columns. Entries (i, j) of the matrix gives the weights w_{ij} from nodes *i* to *j*, which is finite if *i* is linked directly to *j*, and infinite otherwise. The idea of the Floyd's algorithm is straightforward. Given three nodes *i*, *j* and *k* with the connecting weights on three edges, it is shorter to reach *j* from *i* passing through *k* if $w_{ik} + w_{kj} < w_{ij}$.

In this case, it is optimal to replace the direct route from $i \to j$ with the indirect route $i \to k \to j$. This triple operation exchange is applied systematically to the network using following steps:

Step 0: Define the starting weight matrix W_0 and the node sequence matrix S_0 as given below. The diagonal elements are marked with (-) to indicate that they are blocked. Set *k* = 1.

$$W_0 = \begin{array}{c} \\ 1 \\ 2 \\ : \\ i \\ : \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & .... & j & .... & n \end{array} \\ \begin{bmatrix} - & w_{12} & .... & w_{1j} & .... & w_{1n} \\ w_{21} & - & .... & w_{2j} & .... & w_{2n} \\ : & : & : & : & : & : \\ w_{i1} & w_{i2} & .... & w_{ij} & .... & w_{in} \\ : & : & : & : & : & : \\ w_{n1} & w_{n2} & .... & w_{nj} & .... & - \end{bmatrix} \end{array} \text{ and } S_0 = \begin{array}{c} \\ 1 \\ 2 \\ : \\ i \\ : \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & .... & j & .... & n \end{array} \\ \begin{bmatrix} - & 2 & .... & j & .... & n \\ 1 & - & .... & j & .... & n \\ : & : & : & : & : & : \\ 1 & 2 & .... & j & .... & n \\ : & : & : & : & : & : \\ 1 & 2 & .... & j & .... & - \end{bmatrix} \end{array}$$

General Step *k*: Define row *k* and column *k* as pivot row and pivot column. Apply the triple operation to each element w_{ij} in W_{k-1}, for all *i* and *j*.

If the condition $w_{ik} + w_{kj} < w_{ij}$ for different *i*, *j* and *k* is satisfied then implement following changes in W_k and S_k.

- Create W_k by replacing w_{ij} in W_{k-1} with $w_{ik} + w_{kj}$.
- Create S_k by replacing s_{ij} in S_{k-1} with k. Set *k* = *k* +1. If *k*=*n* then stop, else repeat step *k*.

After *n* steps, we can determine the shortest route between nodes *i* and *j* from the matrices W_n and S_n using following rules:

1. From W_n, w_{ij} gives the weights of shortest route between node *i* and *j*.
2. From S_n, determine the intermediate node $k = s_{ij}$ that yield the route $i \to k \to j$.

If $s_{ik} = k$ and $s_{kj} = j$, stop; all intermediate nodes of shortest route has been found, otherwise, repeat the procedure between nodes *i* and *k*, and between *k* and *j*.

3. PROPOSED METHOD

In Fuzzy environment, two issues, namely addition of fuzzy numbers and ranking of numbers should be solved. Based on canonic representation [4], the classical Floyd's algorithm can be easily generalized to a fuzzy Floyd's algorithm as follows:

In proposed method, it is necessary to clarify some basic notations. If $\tilde{a}_{ij}$ is any fuzzy number then a_{ij} represents its crisp form obtained by using graded mean integration representation.

This is a proposed model which determines shortest route between any two nodes in the network under fuzzy environment. The algorithm represents an n - nodes network as a square matrix with n rows and n columns. Entries (i, j) of the matrix gives the weights w_{ij} from nodes *i* to *j* (obtained from fuzzy weights $\tilde{w}_{ij}$ using graded mean integration representation), which is finite if *i* is linked directly to *j*, and infinite otherwise. The idea of the Floyd's algorithm is similar to classical Floyd's algorithm. Given three nodes *i*, *j* and *k* with the connecting fuzzy weights on three edges. It is shorter to reach *j* from *i* passing through *k* if $p_{ij} < w_{ij}$ where p_{ij} is a crisp form of $\tilde{p}_{ij}\left(= \tilde{w}_{ik} + \tilde{w}_{kj}\right)$ obtained by using graded mean integration representation.

In this case, it is optimal to replace direct route from $i \to j$ with the indirect route $i \to k \to j$. This triple operation exchange is applied systematically to the network using following steps:

Step 0: Define the starting weight matrix W_0, where all w_{ij} are crisp values corresponding to fuzzy weights $\tilde{w}_{ij}$ and the node sequence matrix S_0 as given below. Again, the diagonal elements are marked with (-) to indicate that they are blocked. Set *k* = 1.

$$W_0 = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} 1 & 2 & \cdots & j & \cdots & n \end{array} \\ \begin{bmatrix} - & w_{12} & \cdots & w_{1j} & \cdots & w_{1n} \\ w_{21} & - & \cdots & w_{2j} & \cdots & w_{2n} \\ : & : & : & : & : & : \\ w_{i1} & w_{i2} & \cdots & w_{ij} & \cdots & w_{in} \\ : & : & : & : & : & : \\ w_{n1} & w_{n2} & \cdots & w_{nj} & \cdots & - \end{bmatrix} \end{array} \text{ And } S_0 = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} 1 & 2 & \cdots & j & \cdots & n \end{array} \\ \begin{bmatrix} - & 2 & \cdots & j & \cdots & n \\ 1 & - & \cdots & j & \cdots & n \\ : & : & : & : & : & : \\ 1 & 2 & \cdots & j & \cdots & n \\ : & : & : & : & : & : \\ 1 & 2 & \cdots & j & \cdots & - \end{bmatrix} \end{array}$$

General Step *k*: Define row *k* and column *k* as pivot row and pivot column. Apply the triple operation to each element w_{ij} in W_{k-1}, for all *i* and *j*.

If the condition $p_{ij} < w_{ij}$, where $\tilde{p}_{ij}\left(= \tilde{w}_{ik} + \tilde{w}_{kj}\right)$ for different *i*, *j* and *k* is satisfied then implement following changes in W_k and S_k.

- Create W_k by replacing w_{ij} in W_{k-1} with p_{ij}, where $\tilde{p}_{ij}\left(= \tilde{w}_{ik} + \tilde{w}_{kj}\right)$.

- Create S_k by replacing s_{ij} in S_{k-1} with k . Set $k = k$ +1. If k=n then stop, else repeat step k.

After n steps, we can determine the shortest route between nodes i and j from the matrices W_n and S_n using following rules:

3. From W_n , w_{ij} gives the weights of shortest route between node i and j in crisp manner.

4. From S_n , determine the intermediate node $k = s_{ij}$ that yield the route $i \to k \to j$.

If $s_{ik} = k$ and $s_{kj} = j$, stop; all intermediate nodes of shortest route has been found, otherwise, repeat the procedure between nodes i and k, and between k and j.

4. TRANSPORTATION NETWORK APPLICATION

This section illustrates, transportation network under fuzzy environment, where proposed method is used to find shortest route between any two nodes in given network:

Illustration

For the network given in Figure 3, find the shortest route between every two nodes. The transition time are given on edges (refer Table 1), which are fuzzy trapezoidal numbers. Edge $(3,4)$ is directed, so no traffic is allowed from node 4 to 3. All other edges allow two–way traffic.

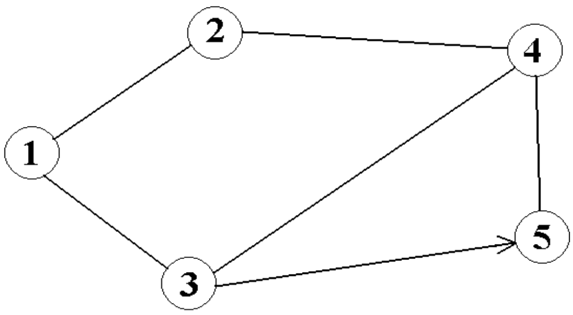

Figure 3: A Transportation Network Model

Table 1: Weights of Edges in the Network Shown in Figure 3

Edge (i, j)	Membership Function $\mu_{\tilde{w}_{ij}}(x)$	$P(\tilde{w}_{ij})$
(1,2)	(1,2,3,5)	2.66667
(1,3)	(5,7,10,11)	8.33333
(2,1)	(1,2,3,5)	2.66667
(2,4)	(2,5,6,7)	5.16667
(3,1)	(5,7,10,11)	8.33333
(3,4)	(3,6,7,8)	6.16667
(3,5)	(11,14,15,17)	14.33333
(4,2)	(2,5,6,7)	5.16667
(4,3)	(3,6,7,8)	6.16667
(4,5)	(1,4,5,7)	4.33333
(5,4)	(1,4,5,7)	4.33333

Solution

<u>**Iteration 0:**</u> The matrices W_0 and S_0 gives the initial representation of the network, where all $(i, j)^{th}$ entries of W_0 are canonical representations of weights of route from *i* to *j*.

W_0

	1	2	3	4	5
1	-	**2.667**	**8.333**	∞	∞
2	**2.667**	-	∞	5.167	∞
3	**8.333**	∞	-	6.167	14.333
4	∞	5.167	6.167	-	4.333
5	∞	∞	100.000	4.333	-

S_0

	1	2	3	4	5
1	-	2	3	4	5
2	1	-	3	4	5
3	1	2	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Iteration 1: Set k = 1. Considering first row and column as pivot row and column in the matrix W_0, one can easily observe that w_{32} and w_{23} satisfies triple operation. Thus W_1 and S_1 are obtained from W_0 and S_0 in the following manner:

1. Replace w_{32} with p_{32} where $\tilde{p}_{32} = \tilde{p}_{31} + \tilde{p}_{12}$ and set $s_{32} = 1$
2. Replace w_{23} with p_{23} where $\tilde{p}_{23} = \tilde{p}_{21} + \tilde{p}_{13}$ and set $s_{23} = 1$

W_1

	1	2	3	4	5
1	-	**2.667**	8.333	∞	∞
2	**2.667**	**-**	**11.000**	**5.167**	∞
3	8.333	**11.000**	-	6.167	14.333
4	∞	**5.167**	6.167	-	4.333
5	∞	∞	∞	4.333	-

S_1

	1	2	3	4	5
1	-	2	3	4	5
2	1	-	1	4	5
3	1	1	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Iteration 2: Set k = 2. Considering first row and column as pivot row and column in the matrix W_1, one can easily observe that w_{41} and w_{14} satisfies triple operation. Thus W_2 and S_2 are obtained from W_1 and S_1 in the following manner:

1. Replace w_{41} with p_{41} where $\tilde{p}_{41} = \tilde{p}_{42} + \tilde{p}_{21}$ and set $s_{41} = 2$
2. Replace w_{14} with p_{14} where $\tilde{p}_{14} = \tilde{p}_{12} + \tilde{p}_{24}$ and set $s_{14} = 2$

W_2

	1	2	3	4	5
1	-	2.667	**8.333**	7.833	∞
2	2.667	-	**11.000**	5.167	∞
3	**8.333**	**11.000**	**-**	**6.167**	**14.333**
4	7.833	5.167	**6.167**	-	4.333
5	∞	∞	∞	4.333	-

S_2

	1	2	3	4	5
1	-	2	3	2	5
2	1	-	1	4	5
3	1	1	-	4	5
4	2	2	3	-	5
5	1	2	3	4	-

By setting k = 3, 4 and 5 and repeating above procedure one can reach to final iteration. In this case up to Iteration 5.

Iteration 5: At last one can have matrices W_5 and S_5 can be obtained as follows, which shows shortest route between any two vertices and one can find out path using matrix S_5 :

W_5

	1	2	3	4	5
1	-	2.667	8.333	7.833	12.167
2	2.667	-	11.000	5.167	9.500
3	8.333	11.000	-	6.167	10.500
4	7.833	5.167	6.167	-	4.333
5	12.167	9.500	10.500	4.333	-

S_5

	1	2	3	4	5
1	-	2	3	2	4
2	1	-	1	4	4
3	1	1	-	4	4
4	2	2	3	-	5
5	4	4	4	4	-

For example, from the matrix S_5 shortest route from node 1 to 5 is 1→2→4→5.

And weight for this path will be $w_{15} = \tilde{w}_{12} \oplus \tilde{w}_{24} \oplus \tilde{w}_{45} \approx 12.167$.

5. CONCLUSIONS

This article extended Floyd's algorithm to find shortest route between any two nodes in a network model with fuzzy weights. Two important issues are discussed. One is how to determine the addition of weights of two edges. The other is how to compare the weights of two different paths when their edges are represented by fuzzy numbers. The proposed method to find shortest route under fuzzy weights is based on graded mean integration representation of fuzzy numbers. A numerical example is used to illustrate the proposed method. This proposed method can be applied in to many real applications such as transportation network, logistic management, traffic control and other network optimization problems where to find shortest route between nodes is a need.

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